

Competition Between Fractional Quantum Hall Liquid, Bubble and Wigner Crystal Phases in the Third Landau Level

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Magnetotransport measurements were performed in a ultra-high mobility GaAs/AlGaAs quantum well of density $\sim 3.0 \times 10^{11} \text{ cm}^{-2}$. The temperature dependence of the magnetoresistance R_{xx} was studied in detail in the vicinity of $\nu = \frac{9}{2}$. In particular, we discovered new minima in R_{xx} at filling factor $\nu \simeq 4\frac{1}{5}$ and $4\frac{4}{5}$, but only at intermediate temperatures $80 \lesssim T \lesssim 120 \text{ mK}$. We interpret these as evidence for a fractional quantum Hall liquid forming in the $N=2$ Landau level and competing with bubble and Wigner crystal phases favored at lower temperatures. Our data suggest that a magnetically driven insulator-insulator quantum phase transition occurs between the bubble and Wigner crystal phases at $T = 0$.

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The fractional quantum Hall effect (FQHE) in a two-dimensional electron gas (2DEG) is typically understood as an incompressible Laughlin liquid of electrons giving rise at low temperatures to a vanishing magnetoresistance, R_{xx} concomitant with a quantized Hall resistance, $R_{xy} = h/\nu e^2$, where ν is the Landau level (LL) filling factor. This effect is now widely described in terms of the composite fermion model (CF) [1, 2, 3, 4], and has been so far observed only in the first ($N=0$) and second ($N=1$) Landau levels. This experimental fact is in agreement with early calculations based on exact diagonalization of a small particle ensemble, which suggest that a Laughlin (or FQHE) liquid is energetically unfavored in the higher Landau levels, $N > 1$ [5, 6, 7]. In this Letter, we report on new magnetotransport measurements performed in a ultra-high mobility GaAs/AlGaAs specimen which show evidence, at intermediate temperatures $80 \lesssim T \lesssim 120 \text{ mK}$, for a FQHE liquid forming in the $N=2$ Landau level at filling factor $\nu \simeq 4\frac{1}{5}$ and $4\frac{4}{5}$.

Previous measurements performed in ultra-high mobility heterostructures have shown, in the lowest Landau level, the CF model to correctly account for the sequence of FQHE of electrons in terms of an integer quantum Hall effect (IQHE) of CF particles. Recent, newly discovered FQHE states such as, for example, at $\nu = \frac{4}{11}$ or the $\nu = \frac{7}{11}$, find in the CF model an elegant interpretation in terms of a FQHE of these flux-attached CF particles [8]. While all FQHE states in the lowest Landau levels are well understood within such a model, the current situation differs much even in the second ($N=1$) Landau level. For example, insulating and re-entrant integer quantized Hall effects (RIQHE) have recently been

reported in the vicinity of $\nu = \frac{5}{2}$ and $\frac{7}{2}$ [9, 10].

The phenomenology observed in higher LL ($N > 1$) is even more complex. Several measurements in these LL have found new and distinct phenomena from those observed in lower LLs. For example, a strong resistance anisotropy is observed at half-integer filling $\nu = \frac{9}{2}$, together with insulating phases existing in its flank [11, 12], giving rise to a RIQHE [13] around filling factor $\nu \simeq 4\frac{1}{4}$ and $4\frac{3}{4}$. These experimental observations certainly point toward many-body correlations of a different lineage than at $\nu < 1$, and in particular are interpreted as ‘stripe’ and ‘bubble’ charge density waves [14, 15, 16, 17, 18] as well as liquid crystalline phases [19]. The question on whether or not a FQHE liquid can form in the $N > 1$ Landau levels has been reexamined by Fogler and Koulakov [20]. Experimental insights on the existence, or non-existence, of a Laughlin liquid in the $N > 1$ LL is therefore timely and highly relevant to our understanding of electron-electron correlations in these higher LLs.

The magnetotransport measurements were performed on a 300 \AA wide modulation-doped GaAs/AlGaAs quantum well, of density $n \simeq 3.0 \times 10^{11} \text{ cm}^{-2}$ and ultra-high mobility $\mu \simeq 27 \times 10^6 \text{ cm}^2/\text{Vs}$. Electrical contact to the 2DEG was achieved via diffusion of small indium beads into the semiconductor surface. The electronic transport was measured at a fixed current varying from 4 to 10 nA, and using a quasi-dc lock-in technique (2-10 Hz). The sample was mounted in the $^3\text{He}-^4\text{He}$ mixture of a top-loading dilution refrigerator. The temperature reported in this work is that of the helium mixture in the mixing chamber, measured with a calibrated ruthenium oxide thermometer; at temperature $T \gtrsim 30 \text{ mK}$, the electron

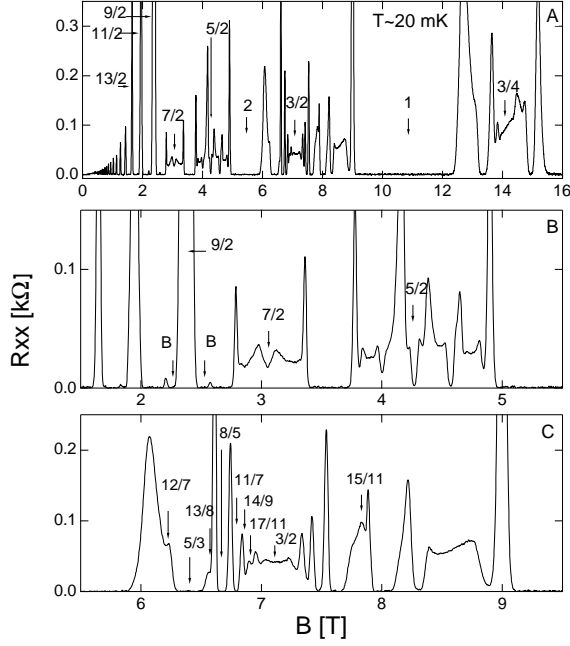


FIG. 1: Longitudinal resistance R_{xx} as a function of magnetic field measured with an excitation current of 4 nA. In the panel A, R_{xx} is shown over the whole field range measured between 0 and 16T, with the principal filling factor denoted with arrows. Panels B and C show the same data, but rescaled such as to show the smaller features. New developing minima in panel C are labeled with their tentative fractional assignment, $\frac{12}{7}$, $\frac{13}{8}$ and $\frac{15}{11}$. The RIQHE state near $\nu = \frac{9}{2}$ is labeled with the letter B.

bath is assumed to be in good thermal contact with the thermometer. The overall accuracy of the thermometry at temperatures above 30 mK (including the correction for the magnetoresistance) is estimated to be within 10% of the value reported.

Before turning to the main subject of the paper, we first describe some salient features observed in this exceptionally high-quality sample. The longitudinal resistance R_{xx} at temperature $T \sim 20$ mK is shown in Figure 1, panel A, over the magnetic field range spanning from 0 to 16T. Enlarged plots for various field ranges are shown in panel B and C to resolve the finer structures. The electronic density was determined from known well-developed FQHE states to be $n = 2.58 \times 10^{11} \text{ cm}^{-2}$. We added, as a guide, several principal filling factors and have shown them as arrows. The minima in R_{xx} near $\nu = \frac{9}{2}$ associated with RIQHE are labeled with the letter B. In terms of the anisotropic $\nu = \frac{9}{2}$ terminology, these data were taken in a contact configuration in the hard direction, i.e. with a maximum in resistance flanking $\nu = \frac{9}{2}$. The exceptional high-quality of the sample can be readily seen near $\nu = \frac{5}{2}, \frac{3}{2}$ where several minima in R_{xx} which are typically observed only in the very best samples are well developed [21].

The series of fractional states observed around $\nu = \frac{3}{2}$ is

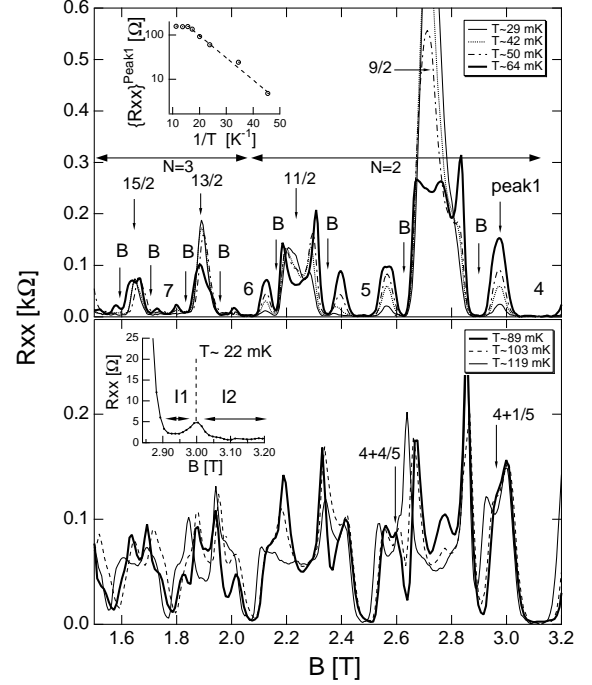


FIG. 2: Magnetotransport R_{xx} at various temperatures. The upper (lower) panel shows the data taken below (above) $T \lesssim 65$ mK. The half-integer filling factors are indicated with arrows. The position of the $\nu = 4\frac{1}{5}$ and $\nu = 4\frac{4}{5}$ fillings are indicated in the lower panel, together with the bubble states in the upper panel, labeled with the letter B. The horizontal arrows indicate the magnetic field range spanned by the $N=2,3$ Landau levels. The upper inset shows an Arrhenius plot of the temperature dependence of the resistance maximum labeled peak1. The inset in the lower panel shows an enlarged version around $\nu = 4\frac{1}{5}$ at $T \sim 22$ mK, with I1 and I2 denoting the two insulating phase.

very well accounted for in the CF model which interpret these as series of FQHE states emanating from $\nu = \frac{3}{2}$ and given by $\nu = (3p \pm 2)/(2p \pm 1)$, where p is the CF filling factor. The filling factors $\frac{5}{3}, \frac{8}{5}, \frac{11}{7}, \frac{14}{9}$ and $\frac{17}{11}$ corresponding to the CF series (+) with $p = 1, 2, 3, 4$ and 5 are shown as arrows in the panel C of Fig.1. In addition to the known states, we also observe three new developing minima in R_{xx} at filling factors $\nu \simeq 1.72$, $\nu \simeq 1.62$, and $\nu \simeq 1.35$, and labeled in Fig.1 with their proposed tentative fractional assignment, $\frac{12}{7}$, $\frac{13}{8}$ and $\frac{15}{11}$, respectively. The small discrepancy between the observed minima and the proposed fractional assignment is believed to be either from the shallowness of these new minima, or from spin effects known to occur in the first excited LL, and understood in terms of a CF model in the presence of a spin [22]. These new minima cannot be explained by the CF series mentioned above. However, the prospect of observing an FQHE at $\frac{15}{11}$ is particularly interesting, for it could be interpreted, similarly to the $\frac{7}{11}$, as a FQHE of CF's [8], but in the upper spin branch of the first LL.

Figure 2 summarizes our most remarkable findings in

the $N=2,3$ LL. The magnetotransport between $\nu = \frac{9}{2}$ and $\nu = \frac{15}{2}$ was studied in detail at various temperatures. The higher electron density $n = 3.02 \times 10^{11} \text{ cm}^{-2}$ than in the data of Fig.1 results from different illumination conditions with a light emitting diode (LED) during the cooldown of the sample to low temperatures. Of particular interest in the upper panel is the existence of minima, B, in the flank of the half-integer filling factors $\nu = \frac{9}{2}, \frac{11}{2}, \frac{13}{2}$ and $\frac{15}{2}$. These minima are located approximately at filling factor $\nu^* \equiv \nu - [\nu] \simeq \frac{1}{4}$ and $\frac{3}{4}$, where $[\nu]$ is defined as the largest integer smaller than ν , and have been previously observed in DC transport by Lilly *et al.*[11] and Du *et al.*[12], and more recently as a microwave resonance by Lewis *et al.*[23]. These RIQHE were interpreted in terms of a triangular crystal lattice phase with at least two electrons per lattice site, the so-called ‘bubble’ phase [14]. The data in the upper panel of Fig.2 represents a particularly good case of the ‘bubble’ phase persisting up to the $N=3$ LL.

The temperature dependence in the vicinity of $\nu = \frac{9}{2}$ is particularly interesting, for we observe a crossover from a maximum (peak) to a minimum (dip) of resistance, at $T \sim 65$ mK. A similar crossover is also observed in the temperature dependence of the resistance peak separating the bubble state at $\nu^* \sim \frac{1}{4}, \frac{3}{4}$ and the IQHE at $\nu = 4, 5$, respectively. For these resistance maxima, a strong temperature dependence is observed at $T \lesssim 65$ mK, while for greater temperatures the resistance peak remains constant and new structures are forming in the vicinity of the peaks, which we discuss below in details. The temperature dependence of the resistance peak labeled ‘peak1’ is explicitly given in the inset of the upper panel of Fig.1 in an Arrhenius plot. The dotted line is a fit to the lower temperature data which show an activated behaviour $R_{xx}^{peak1} \propto e^{-E/T}$ in the range $15 \lesssim 1/T \lesssim 50 \text{ K}^{-1}$, with a characteristic energy scale $E \sim 0.11(1) \text{ K}$. At temperature above ~ 65 mK, (or $1/T \sim 15 \text{ K}^{-1}$) the saturation of the resistance signals the onset of a different regime. The inset in the lower panel of Fig.2 shows an enlarged plot of the resistance maximum separating the bubble phase[13, 23] from the IQHE, at the lowest temperature reached in this experiment, $T \sim 22$ mK. A small residual resistance peak, of amplitude $\sim 5 \text{ Ohm}$ separates the two insulating phases, labeled I1 and I2. Previous micro-wave conductivity measurements found distinct resonances in both insulating phases, and interpreted these in terms of bubble (I1) and Wigner crystal (I2) phases, respectively [23, 24, 25]. The data in Fig.2 suggest that $R_{xx}^{peak} \rightarrow 0$ as $T \rightarrow 0$, opening up the interesting possibility of a magnetically driven insulator-insulator quantum phase transition between bubble and Wigner crystal phases at $T = 0$.

Recent microwave measurements in the same filling factor region as the position of the resistance peaks separating bubble and Wigner crystal phases found evidence for a coexistence region between the two insulating

phases at $T \sim 35$ mK[25]. On the basis of these measurements, we interpret our finite resistance peak near $\nu^* \simeq \frac{1}{5}, \frac{4}{5}$ as originating from poorly localized electronic states forming in this mixed insulator phase region. In the limit $T \rightarrow 0$, all electrons in the uppermost Landau levels are fully localized, and ordered into either Wigner (I2) or bubble (I1) crystal phases. As the temperature is increased, conduction channels open up as the electrons at the boundary between I1 and I2 delocalize. These electronic excitations which show an activated behaviour contribute to the diagonal component of the conductivity tensor, $\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2)$, thus giving rise to the non-zero resistance observed, $R_{xx} \propto \rho_{xx}$. The change in temperature dependence of R_{xx}^{peak} together with the appearance of new structures/minima located in the peaks, suggest that these unlocalized electronic states may give rise to a FQHE at temperatures $T \gtrsim 65$ mK.

The observation of new structures/minima in R_{xx} at filling factor near $\nu = 4\frac{1}{5}$ and $\nu = 4\frac{4}{5}$ is striking, for it suggests that an incompressible Laughlin liquid may be able to form in the $N=2$ LL. These two new structures, shown in the lower panel of Fig.2, reside on the resistance peaks separating bubble and Wigner crystal phases, and are only observed at intermediate temperatures, $80 \lesssim T \lesssim 120$ mK. We regard these minima as the first experimental evidence of a FQHE liquid forming in the third Landau level.

The disappearance of the minima at lower temperatures can be interpreted as a competition with I1 and I2 below $T \sim 65$ mK, in a manner reminiscent of the FQHE states observed at very small filling, $\nu < \frac{1}{5}$, and obeying similar energetics[26]. Combining our observations with that of Lewis *et al.*[25] suggest the following phase diagram in the third Landau level (lower spin branch). At finite temperatures in the filling factors region encompassing the resistance peaks observed at $4\frac{1}{5}$ and $4\frac{4}{5}$, there is simultaneous coexistence of bubble and Wigner crystal phases, together with thermally activated delocalized electronic states present in the same region. At sufficiently high temperatures, an excited fractional quantum Hall liquid forms in this coexistence phase, but yields to the electron solid phases at lower temperatures as the number of delocalized states decreases. The resistance peaks observed at $4\frac{1}{5}$ and $4\frac{4}{5}$ then disappear with $T \rightarrow 0$ as a consequence of complete localization of the electronic states into Wigner and bubble electron solids, the $T = 0$ ground state.

A novel many-body ground state based on charge density waves in weak magnetic fields, and with the uppermost Landau level $N \gg 1$ partially filled, has been proposed by Koulakov *et al.*[15] and independently by Moessner and Chalker[16]. Subsequently, Fogler and Koulakov[20] examined the energetics of various many-electron states and showed that a transition (as a function of N) occurs at $\nu^* = \frac{1}{3}$ from a Laughlin to a charge density wave in Landau levels $N \geq 2$. Interestingly, their

analysis showed $\nu^* = \frac{1}{5}$ to be the only filling able to form a FQHE liquid in the $N=2$ LL. Similarly, recent calculations from Goerbig *et al.* showed the energetics of the $\nu^* = \frac{1}{5}$ quantum liquid to be favored in the $N=2$ LL, and very close to a mixed phase of one- and two-electron bubbles [27]. However, given that the finite size effects (z-extent of the electronic wave function) in the quantum well may substantially change these energetics, and that the difference in energies at $\nu^* = \frac{1}{5}$ between Laughlin liquid and charge density wave is so small, an unambiguous determination of the lowest energy state based on such calculations is not possible.

Nevertheless, our results in the $N=2$ LL lend themselves very well to this interpretation. Importantly, the data show evidence for a FQHE liquid *only* at filling factor $\nu^* = \frac{1}{5}, \frac{4}{5}$ in the $N = 2$ LLs. Within the theoretical scenario discussed above, the disappearance of these FQHE states at temperatures, $T \lesssim 80$ mK, would result from a competition between a FQHE state and another phase present at the bubble and Wigner crystal phase boundary.

To summarize, we have performed magnetotransport measurements in an ultra high-mobility GaAs/AlGaAs specimen which showed resistance minima associated with re-entrant integer quantum Hall effects in the $N = 2, 3$ LLs. The data suggest that a magnetically driven insulator-insulator phase transition may occur at $T = 0$. Striking new minima are observed at $\nu^* \simeq \frac{1}{5}, \frac{4}{5}$ in the $N=2$ Landau level, but only at intermediate temperatures, $80 \lesssim T \lesssim 120$ mK. This may signal a possible competition between a Laughlin liquid and another phase at lower temperature, and support theoretical scenarios based on the premisses of a charge density wave in the ground state of $N > 1$ Landau levels and in weak magnetic fields.

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